Teacher Education and Development Study in Mathematics 2008





Conseil des ministres de l'Éducation (Canada)

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## CANADIAN REPORT





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## Table of Contents

| What is TEDS-M 2008?  | 1  |
|---|----|
| Research design and methodology for TEDS-M  | 2  |
| What is to be learned from TEDS-M?  | 2  |
| What value will TEDS-M add to existing research on teacher education?   | 3  |
| Target populations  | 3  |
| Components of TEDS-M  | 4  |
| Component 1: studies of teacher-education policies for<br>elementary and lower-secondary mathematics<br>teachers and their cultural and social contexts         | 4  |
| Component 2: studies of elementary and lower-secondary<br>mathematics teacher-preparation routes, programs,<br>standards, and expectations for teacher learning | 5  |
| Component 3: studies of the mathematics and related<br>teaching knowledge of future elementary<br>and lower-secondary school teachers                           | 7  |
| Participating countries in TEDS-M   | 9  |
| The Canadian administration   | 9  |
| Structure and organization of teacher education in Canada   | 10 |
| Programs in specific jurisdictions  | 12 |
| Ontario   | 12 |
| Quebec  | 12 |
| Nova Scotia   | 13 |
| Newfoundland and Labrador   | 14 |

| The administration of the TEDS-M survey in Canada  | . 14 |
|--|------|
| Representation at international meetings   | . 15 |
| Adaptation, translation, and approval of assessment and questionnaire materials  | . 15 |
| Sampling institutions  | . 15 |
| Contacting institutions and sending and collecting materials   | . 16 |
| Quality monitoring   | . 16 |
| Coding   | . 17 |
| Description of teacher-education courses and collection and coding of syllabi  | . 17 |
| Coding of mathematics content knowledge and mathematics pedagogical content knowledge  | . 17 |
| Results  | . 18 |
| Institutional Program Questionnaire  | . 18 |
| Survey for Mathematics, Mathematics Pedagogy, and<br>General Pedagogy Educators  | . 19 |
| Survey for Future Teachers of Lower-Secondary Mathematics  | . 21 |
| Survey for Future Teachers of Elementary Mathematics   | . 22 |
| Conclusion   | . 30 |
| References   | 31   |
| APPENDIX I   | . 33 |
| Sample Syllabus Coding   | . 33 |
| APPENDIX II  | . 35 |
| Examples of assessment items in the Primary General Pedagogy<br>Knowledge for Teaching section of the Future Teacher Questionnaire   | . 35 |
| Examples of assessment items in the Secondary General Pedagogy<br>Knowledge for Teaching section of the Future Teacher Questionnaire | . 41 |

## List of Figures

| Figure 1 | Mean scores and confidence intervals for participating<br>countries—Elementary-level future teachers' mathematics<br>content knowledge      | 24 |
|----------|---|----|
| Figure 2 | Mean scores and confidence intervals for participating<br>countries—Elementary-level future teachers' pedagogy<br>content knowledge         | 25 |
| Figure 3 | Mean scores and confidence intervals for participating<br>countries—Lower-secondary-level future teachers'<br>mathematics content knowledge | 26 |
| Figure 4 | Mean scores and confidence intervals for participating<br>countries—Lower-secondary-level future teachers'<br>pedagogy content knowledge    | 27 |
| Figure 5 | Mean scores and confidence intervals by gender:<br>primary mathematics knowledge  | 28 |
| Figure 6 | Mean scores and confidence intervals by gender:<br>primary pedagogical knowledge  | 28 |
| Figure 7 | Mean scores and confidence intervals by gender:<br>lower-secondary mathematics knowledge  | 29 |
| Figure 8 | Mean scores and confidence intervals by gender:<br>lower-secondary pedagogical knowledge  | 29 |

## What is TEDS-M 2008?

The Teacher Education and Development Study in Mathematics 2008 is a comparative study of teacher education, with a focus on the preparation of mathematics teachers at the elementary and lower-secondary levels. The study was carried out under the aegis of the International Association for the Evaluation of Educational Achievement (IEA), a consortium of research institutions in 60 countries. TEDS-M focused particularly on the links between teacher-education policies, practices, and outcomes. The study provided participating countries with a valuable opportunity to conduct research on their own teacher-education systems and to learn from approaches used in other countries.

Teacher education has become an area of considerable interest among policy-makers in many countries over recent years. This reflects the growing body of research on the vital importance of teachers' knowledge and skills to quality learning opportunities for students. It also reflects the need to recruit and prepare a new generation of teachers as large numbers of current teachers reach retirement age.

TEDS-M addressed research questions of central interest to policy-makers who want to improve the effectiveness of teacher-education systems, such as:

- What are the characteristics of teacher-education programs that effectively prepare future mathematics teachers?
- What kinds of learning experiences are effective in transforming the beliefs of future teachers about the teaching and learning of mathematics?
- What kinds of school experiences are most effective in preparing mathematics teachers?
- How can the outcomes of teacher-education programs for mathematics teachers be measured in ways that are reliable and valid?
- Under what conditions can national policies for the regulation or accreditation of teacher education have a positive impact on the quality of outcomes from teacher education?
- What kinds of policies are proving to be effective in recruiting mathematics teachers from a diverse range of social and cultural backgrounds?

The main goal of TEDS-M was to show whether and how much teacher-preparation policies, programs, and practices across the world contribute to the capability to teach mathematics effectively in elementary and lower-secondary schools. The TEDS-M project included:

- studies of country context and of teacher-training policies, programs, and practices at the national level;
- studies of curricula and teacher-preparation practices, including standards and expectations for future teachers, at the institutional level;
- studies of the impact of teacher preparation on the knowledge, skills, and dispositions acquired by future teachers.

## Research design and methodology for TEDS-M

The study surveyed future teachers at the end of their teacher preparation in order to examine elementary and lower-secondary teachers' mathematical knowledge and its relationship with future teachers' opportunity to learn.<sup>1</sup>

TEDS-M used a national probability sample and a customized sample plan for each participating country.

One of the fundamental principles of IEA design is to base cross-national comparisons on national probability samples. This approach entails the development of an international master sampling plan, the adaptation of this master plan to national contexts, and the adjudication of differences by an IEA-designated sampling referee. TEDS-M sampled teacher-education institutions, instructors, and future teachers.

## What is to be learned from TEDS-M?

• TEDS-M provided answers on the POLICY and CONTEXT of mathematics teacher education

TEDS-M examined the intended and implemented policies that support elementary and lower-secondary teachers' achieved level and depth of mathematics and related teaching knowledge, and how teacher-education policies influence the structure of elementary and lower-secondary mathematics teachers' opportunities to learn.

• TEDS-M provided answers on the ORGANIZATION of mathematics teacher education

TEDS-M examined the learning opportunities available to future elementary and lower-secondary mathematics teachers that allow them to attain the necessary knowledge to teach mathematics. It also studied the structure of the learning opportunities, the content of teacher-education programs, and the organization of instruction.

• *TEDS-M provided answers on the OUTCOMES of mathematics teacher education* TEDS-M examined the level and depth of the mathematics and related teaching knowledge attained by future elementary and lower-secondary teachers, and how this knowledge varies across countries.

<sup>&</sup>lt;sup>1</sup> The two target populations for future teachers were those teaching at the primary/elementary level and the lower-secondary level. In Canada, this corresponds to Grades 1 to 6 and Grade 8 (Secondary II in Quebec).

# What value will TEDS-M add to existing research on teacher education?

TEDS-M is the first mathematics teacher-education study to follow a rigorous methodology to:

- gather empirical data on the experience of participating countries to contribute to policy debate about the nature, benefits, and costs of teacher education;
- strengthen the knowledge base to address participating countries' national priorities;
- develop concepts, measurement strategies, indicators, and instrumentation to strengthen the research in this field;
- further a more scientific approach to the study of teacher education and teacher learning in mathematics.

## Target populations

The target populations of TEDS-M included:

- education policy-makers: individuals or organizations responsible for the development and implementation of national (provincial/regional) policies for teacher education, including recruitment, curriculum, quality assurance, and funding;
- teacher-education institutions in participating countries: secondary or postsecondary schools/colleges/universities that offer structured programs of study on a regular and frequent basis to future teachers within a system of teacher preparation;
- educators of future teachers: persons with regular, repeated responsibility to instruct or mentor future teachers within a given teacher-preparation program;
- future teachers: persons enrolled in a teacher-preparation program that is explicitly intended to prepare individuals to teach mathematics in any grade at the elementary or lower-secondary school level.

TEDS-M had three components:

## Component 1:

studies of teacher-education policies for elementary and lower-secondary mathematics teachers and their cultural and social contexts

## **OBJECTIVE:**

To examine the *intended*, *implemented*, and *achieved* policies directed at mathematics teachers, including recruitment, selection, preparation, and certification

## QUESTIONS:

- a) What are the policies that regulate and influence the design and delivery of mathematics teacher education for elementary and lower-secondary teachers within and across countries?
- b) How do countries' distinctive political, historical, and cultural contexts shape mathematics teaching and learning, and how do these influence policy and practice in mathematics teacher education?
- c) What are the consequences of policies for the development of standards for degrees, coverage of topics, and certification practices? What are the consequences of policies for the recruitment, selection, and preparation of future mathematics teachers? Are these policies coherent or do they compete with one another?
- d) What would it take (in terms of resources, capacity, and costs) to bring the preparation of average mathematics teachers to a level that produces high-quality elementary and lower-secondary mathematics teachers in participating countries? What are the comparative costs of implementing alternative teacher-education policies in different contexts?

## DATA-COLLECTION INSTRUMENTS:

These studies used a set of guidelines and protocols, in conjunction with interviews and focus groups of policy-makers in teacher education, to examine:

- documents reflecting national-level policy regarding the teaching of mathematics underlying each route (i.e., consecutive or concurrent teacher education);
- descriptions of the mathematics curricula for the different teacher-education programs within routes;
- descriptions of elementary and secondary school curricula, standards, and examinations;
- the implementation of teacher policy (recruitment, selection, and certification) within each route and institution; and
- the costs of alternative mathematics teacher education for future elementary and secondary teachers.

These data revealed national teacher-preparation policies and trends; how national authorities define a qualified mathematics teacher; their implicit or explicit theories about how one becomes qualified; the kinds of policies that produce enough highly qualified mathematics teachers; and the continuity of these policies from development to implementation. Because of the complexity entailed in defining a route in countries with a decentralized or federalized governance structure, specific states or provinces were selected, with the recognition that policy structures vary widely by state/province, and that even within a state/province, alternative pathways to credentialing vary as well.

## Component 2:

studies of elementary and lower-secondary mathematics teacher-preparation routes, programs, standards, and expectations for teacher learning

#### **OBJECTIVE:**

To examine the *intended* and *implemented* curricula in teacher education

#### QUESTIONS:

- a) What kinds of institutional and field-based opportunities are provided to future elementary and lower-secondary mathematics teachers?
- b) How are program expectations, curricula, and standards enacted?
- c) What are the qualifications and prior experiences of the university mathematics lecturers/ professors/instructors and teacher educators responsible for the implementation of these programs?
- d) What factors explain how much impact routes, programs, and practices have on the mathematics knowledge of future mathematics teachers?

#### DATA-COLLECTION INSTRUMENTS:

#### 1. Institutional Questionnaire on Teacher-Preparation Programs

This questionnaire asked questions regarding the nature and depth of mathematics taught to future teachers in general education and in the professional curriculum; the kind of program future mathematics teachers followed (i.e., consecutive or concurrent); the standards for teacher preparation in mathematics; the kinds of assessments and the level of performance required; and the pedagogical preparation and mathematics-specific pedagogical preparation. It also included questions on the resources used to operate teacher-education programs, as well as on the general profile of teacher educators (e.g., credentials, professional path, courses taught). For countries that had a concurrent route, only future teachers in the last year of teacher education were surveyed, and their knowledge of mathematics was approximated from the study of the national curriculum.

## TEDS-M used a national probability sample of teacher-education institutions in each country

In countries where institutions offer both elementary and secondary education, one sample combining elementary and secondary schools was chosen for each route. In cases where institutions offer more than one route, the frame was organized by the number of routes an institution offered. Some countries opted to select a cohort of students at the beginning of their teacher education. In that case, the larger of the two cohorts was used as the measure of size.

*Target population:* secondary or postsecondary institutions that offered opportunities to learn (OTL) to future elementary and secondary mathematics teachers within a major route.

*Sampling frame:* routes that could lead to elementary and secondary school teaching were identified. Within each route, eligible institutions within the target population were identified, with some measure of size (e.g., number of future elementary and/or secondary school teachers in their final year's cohort). If the figures for elementary and secondary grades were difficult to obtain, more readily available figures were used for the total size of these populations at the elementary or lower-secondary levels, including the required stratification variables.

*Sample design:* stratification was done by additional attributes, such as type of institution, urban/rural setting, sub-national region, or administrative jurisdiction. The selection was made using a stratified probability proportional to size (PPS) systematic sample design. Some national conditions made this design choice impractical, in which case the national design was approved by the sample referee, in agreement with the TEDS-M Joint Management Committee.

*Sample size:* at least 50 institutions from each route were sampled. Countries with much smaller numbers of institutions conducted census sampling. Countries with much larger numbers of institutions sampled institutions.

## 2. Questionnaire for Mathematics Instructors and Teacher Educators

This questionnaire collected data from mathematics instructors and teacher educators on their background, their mathematics-teaching knowledge, the materials used in their instruction, their beliefs about mathematics, and their expectations for future teachers. Parts of this questionnaire were similar to the Future Teacher Questionnaire (see below) in order to examine correspondence and differences between faculty and future teachers (on such things as knowledge, pedagogy, and beliefs) as an indicator of coherence between these two populations.

## TEDS-M sampled mathematics instructors and teacher educators

If the sub-sampling of instructors was deemed required or appropriate, a list of eligible instructors was drawn and a sample plan was submitted for acceptance.

*Target population:* persons with regular, repeated responsibility to instruct or mentor future elementary and lower-secondary mathematics teachers within a given route

Sampling frame: a list of eligible instructors at each sampled teacher-education institution with a minimum of four strata was established, as follows: (1) instructors of mathematics whose primary responsibility is to teach the content of mathematics; (2) teacher educators whose primary responsibility is to help students understand the pedagogy and learning of mathematics in elementary and/or secondary schools; (3) other teacher educators in an academic component; and (4) other teacher educators in a field-experience component. These four strata applied to instructors of both elementary and lower-secondary future teachers, when appropriate.

*Sample design:* all eligible instructors at sampled institutions were to be surveyed. However, some national conditions made this design choice impractical, in which case the national design was approved by the sample referee, in agreement with the TEDS-M Joint Management Committee.

*Sample size:* in nearly all institutions, it was expected that the number of eligible instructors would be too small for sub-sampling in at least one, and frequently more than one, of the strata. Sub-sampling was used only when the number of respondents within the stratum of a sampled institution was larger than needed and overly burdensome to survey.

## 3. Content Analysis of the Teacher-Education Mathematics Curriculum

A protocol was developed to analyze syllabi and sample assignments from teacher-education mathematics curricula in relation to mathematics standards for elementary and secondary students in participating countries and to the Trends in International Mathematics and Science Study (TIMSS) international database of mathematics content standards. In addition, the protocol examined the relation between the content and performance expectations of courses in mathematics teacher-education curricula, and the local or national examinations for teacher certification or licensing. These analyses produced an initial profile of the implemented curricula in mathematics teacher education in terms of the knowledge, pedagogy, dispositions, and other knowledge that future teachers are exposed to as they get ready to teach.

## Component 3:

studies of the mathematics and related teaching knowledge of future elementary and lower-secondary school teachers

#### **OBJECTIVE:**

To examine the *intended* and *achieved* outcomes of teacher education, focusing on answering the following research questions:

#### QUESTIONS:

a) What is the content knowledge of mathematics that future teachers are expected to acquire in the participating countries?

- b) What is the depth of understanding that they are intended to attain?
- c) What beliefs about the teaching and learning of mathematics are promoted by teachereducation programs? What beliefs do future teachers hold about teaching and learning mathematics at the end of their preparation?
- d) What is the knowledge for mathematics teaching (e.g., of the content, pedagogy, curriculum, and attitudes) that future elementary and lower-secondary mathematics teachers actually have at the end of their teacher preparation and once they are considered "ready to teach"?
- e) What other characteristics help to explain future mathematics teachers' ability to master this knowledge?

## DATA-COLLECTION INSTRUMENT:

#### Future Teacher Questionnaire

This questionnaire measured the *intended* and *achieved* mathematics and teaching-related knowledge and beliefs among future teachers.

The questionnaire asked future teachers about their:

- academic background
- mathematics content knowledge
- knowledge of mathematics-specific pedagogy and of general pedagogy
- beliefs concerning students' dispositions and abilities when learning mathematics
- beliefs about purposes for learning mathematics
- self-perception of their level of preparedness to teach mathematics effectively

## TEDS-M used a national probability sample of future teachers

The sample was distributed among the selected teacher-education institutions as evenly as possible. Some countries opted to sample from both beginning and ending cohorts. Countries that opted to allow for sub-national comparisons ensured that an effective sample size was selected from each of the sub-national domains of interest, and adjusted their total sample size accordingly.

*Target population:* Future teachers were defined as all members of a route, starting from the point at which they were enrolled in a formal OTL explicitly intended to prepare teachers to teach mathematics in any of the grades included in the elementary and secondary levels, in their last year of teacher education.

*Sampling frame:* A list of eligible students in their last year of teacher education from each sampled institution within identified routes was established. In institutions that prepared teachers for both elementary and lower-secondary schools, both domains were sub-sampled.

*Sample design:* The simplest approach was used, an equal-size simple random sampling of eligible future teachers from the sampled institutions.

*Sample size:* An effective national sample size of 400 future elementary and secondary teachers for each route was desired.

## The TEDS-M final report is bringing together the three components

The route and institutional profiles of teacher education, the institutional questionnaire, the curriculum analysis, and the instructor/teacher-educator data will provide an institutionallevel as well as a larger contextual frame for the analysis of program data collected in this study. These data will help to refine key concepts and clarify the theorized coherence in order to facilitate further analysis across and within programs as they influence teacher knowledge (via curriculum and instruction). The data collected from the Future Teacher Questionnaire are correlated with the institutional data in order to construct profiles of the *intended*, *implemented*, and *achieved* curriculum for mathematics teachers by country, route, and institution.

## Participating countries in TEDS-M

- Botswana
- Canada
- Chile
- Georgia
- Germany
- Malaysia
- Norway
- Oman
- Philippines
- Poland
- Russian Federation
- Singapore
- Spain
- Switzerland
- Taiwan (Chinese Taipei)
- Thailand
- United States

## The Canadian administration

In Canada, education is the exclusive responsibility of provinces and territories. Thus, there is no national system of teacher education and certification in the country, and individual jurisdictions determine whether they wish to participate in international education-related projects such as TEDS-M.

Four Canadian provinces chose to participate in TEDS-M: Newfoundland and Labrador, Nova Scotia, Ontario, and Quebec. Six of Ontario's 13 institutions participated; six of Quebec's 12 institutions participated; two of Nova Scotia's four institutions participated; as did the single institution in Newfoundland and Labrador. The Canadian administration was carried out between March 7 and April 30, 2008. An exception was made for one institution to accommodate the length of its students' practicum; therefore, that administration was carried out in May.

The sample was drawn randomly from a list of all possible participants supplied by each institution. A sample of 40 future teachers was drawn for each institution; their professors were selected to complete the survey.

## Structure and organization of teacher education in Canada

Teacher education is offered at a total of 56 institutions in Canada. A small number of these are affiliates of larger institutions and include English and French programs within the same institution; for program purposes, English and French programs within one institution are considered independent units. The total number of institutions has not changed much in recent years. Multiple institutions are found in all but two provinces, Newfoundland and Labrador and Prince Edward Island. There are no programs in Canada's three territories, as they tend to draw their teachers from the provincial teacher-education institutions across the country.

Approximately 50,000 students are enrolled in teacher-education programs each year (Statistics Canada, 2008). The enrolment trend over time has been toward a slight decline in the 1990s, followed by a slight increase since 2000. The ratio of females to males is more than 3:1, and there has been a slight increase in this ratio in recent years. Just under 40 per cent of those enrolled, or about 18,000 students, graduate in any one year, a figure that has remained fairly constant in recent years. However, due to the range of one-year and multi-year education programs offered, this percentage cannot be taken as a direct measure of productivity.

Programs range widely in size, enrolling from fewer than 100 to almost 4,000 students, and graduating from about 30 to more than 1,200 students per year. The number of faculty members ranges from as low as 6 to more than 150 per institution. Program size appears to be linked to program variety and specialization. For example, programs in small institutions may have a liberal-arts focus, based on the argument that a general program in liberal arts and the foundations of education is appropriate preparation for teaching at any level. Such universities tend to consider their small size a virtue, and in their promotion, they tend to highlight their small classes, the close ties between faculty and students, and their focus on liberal arts.

In general, there are two routes to graduation at teacher-education institutions — concurrent or consecutive. Concurrent programs usually offer four years of both professional education courses along with academic courses. Some of these concurrent programs lead to the Bachelor of Education (B.Ed.) degree; others, requiring five years of study, lead to a degree in an academic specialty as well as a B.Ed. Consecutive programs require candidates to first obtain an academic degree in order to be accepted into a program of studies in education, usually concentrated in one or two years (two to four semesters). The duration is related to certification requirements; for example, a two-year program following a first degree is the minimum requirement for certification in Nova Scotia, while a one-year program is required for certification in Ontario.

Some institutions offer only one route, while others offer both routes. The general trend across most provinces is toward consecutive programs. The exception is Quebec, where almost all programs offer concurrent studies.

Because most jurisdictions have strong incentives for upgrading academic credentials built into their salary scales, most teachers continue their academic education beyond the first degree. For this reason, it is common for teachers to possess two undergraduate degrees, one in an area of academic specialization and a B.Ed. or equivalent. In any event, two degrees is the normal outcome of consecutive teacher-education programs. Across Canada, close to 20 per cent of teachers have a master's degree, usually in education. The proportion holding advanced degrees varies across jurisdictions and is linked to the structure of the certification/salary scale.

In either of the two routes to a B.Ed., most institutions offer programs at two levels, elementary and secondary. Teachers of the elementary grades are usually considered generalists, while teachers at the secondary level are expected to specialize in one or more disciplines. Consequently, most elementary programs are concurrent and most secondary programs are consecutive. The specialization associated with an academic degree in a major subject area is considered desirable for secondary teachers. The concept of a "teachable subject," that is, a discrete subject in the provincial curriculum, is often transferred from certification requirements to academic major subjects, creating the expectation that the academic major is some subject that can be taught in schools.

In some jurisdictions, teaching certificates are endorsed only for specific levels or subjects. However, the degree to which the teachers holding these endorsed certificates are restricted to their defined areas of specialization varies by jurisdiction and location and depends on teacher supply and demand.

Beyond the two program routes and the two levels for which teachers are prepared, a variety of more specialized programs can be found. For example, physical education and music are often separated from mainstream programs and offered either through the respective academic units or through collaborative arrangements between programs. Special education, second-language teaching, Aboriginal studies, and technological studies are other examples of areas in which specialized programs have been developed. Sometimes, these are organized as separate degrees (such as a Bachelor of Special Education or a Bachelor of Physical Education), but these areas are also specializations within the mainstream degrees.

All teacher-education programs require student teachers to participate in some in-school teaching experience, referred to variously as a practicum, an internship, or student teaching. Indeed, this is generally a significant component of teacher-education programs. The long-term trend is toward longer in-school placements, distributed throughout the program, rather than concentrated at the end.

TEDS-M from the International Association for the Evaluation of Educational Achievement (IEA) was conducted in four Canadian jurisdictions — Ontario, Quebec, Nova Scotia, and Newfoundland and Labrador.

## ONTARIO

There are 14 publicly assisted faculties of education in Ontario universities; together, they enrol approximately 13,000 students and graduate about 8,000 teacher candidates per year. All of the publicly assisted faculties of education offer programs in English, and two offer programs in French.

Almost all Ontario institutions offer consecutive programs, of two semesters' duration, to students who already have a bachelor's degree. The practicum ranges from a minimum of 42 days to over 60 days, depending on the institution. The number of graduates from teacher-education programs ranges from fewer than 100 per year to more than 1,000 per year in the larger institutions, although most programs enrol at least several hundred students. A three-level structure, categorized into Primary/Junior (Grades K–6), Junior/ Intermediate (Grades 4–10), and Intermediate/Senior (Grades 7–12), is typical, and this structure conforms to the structure for teacher certification. This allows teachers to be certified to teach a range of grade levels. Some, more highly specialized, programs are offered in areas such as music, second-language teaching, and technological studies. Candidates for admission to the faculties of education in Ontario apply through the central Teacher Education Application Service (TEAS), which allows applicants three choices of institution or program. Individual universities conduct their selection of candidates.

Teacher certification in Ontario is governed by an independent body, the Ontario College of Teachers (OCT). The governing council of this body comprises 23 teacher-members, elected by members of the college, and 14 members of the public, appointed by the provincial government and broadly representative of other stakeholders in education. Graduates of teacher-education programs at Ontario universities apply to OCT for a general certificate of qualification and registration as their licence to teach. OCT's "Transition to Teaching" studies indicate that, early in the decade, the province was experiencing a general shortage of teachers. However, in recent years, Ontario's employment market has been characterized by an oversupply of teachers in most subject areas as the number of new teachers entering the profession has substantially increased and the number of teachers leaving the profession or retiring has declined (Ontario College of Teachers, 2009).

#### QUEBEC

Twelve institutions offer teacher-education programs in Quebec, nine in French and three in English. Close to 18,000 students are enrolled in these programs, and about 4,000 graduate per year. This rate reflects the concurrent nature of most Quebec programs, under which students take four years to complete the B.Ed. degree. Typically, programs are divided into elementary and secondary levels, with many more specialized programs. Candidates for teaching at the elementary level are prepared as generalists; candidates for teaching at the secondary level are expected to specialize in subjects or subject clusters corresponding to those taught in secondary schools. Most programs are specifically designed to prepare candidates for certification as teachers in Quebec.

Teacher certification in Quebec is governed by the *Comité d'agrément des programmes de formation à l'enseignement (CAPFE)* [committee on accreditation of teacher-training programs], a representative body of stakeholders. CAPFE is responsible for accrediting teacher-training programs in the province, and it is an autonomous, independent body created and funded by the Ministry of Education, Recreation and Sports.

The most recent work on teacher supply and demand in Quebec was a report by the *Service des études économiques et démographiques* [department of economic and demographic research] dated November 2004. The report (Ministère de l'Éducation du Québec, 2004) noted that teacher shortages were expected for teachers of French language arts and social sciences in the period from 2005 to 2007 due to changes to the curricula. There would also be shortages in 2006 in the number of second-language (French and English) teachers, arts teachers, and physical-education teachers. It was also expected that there would be a deficit in mathematics and science teachers in French-language schools. The report also predicted that declining enrolment in schools would lead to a major surplus of teachers in all disciplines for 2008 to 2013.

### NOVA SCOTIA

Four institutions in Nova Scotia offer teacher-education programs, three in English and one in French. All these programs are relatively small compared to national averages, with approximately 900 students and 400 graduates per year in total. This represents a substantial decrease from the early 1990s, when an external review led to the closure of programs at several institutions and a reduction in the number of students accepted into teacher education. That review also led to the establishment of the only system in Canada in which a two-year (four-semester) consecutive program is the norm and a requirement for certification.

Nova Scotia is one Canadian jurisdiction in which high-profile, government-commissioned, external reviews have had an impact on teacher education. The most recent review, completed in 2008 (Nova Scotia Department of Education, 2008), addressed both program structures and labour-market issues. The major labour-market concern was the continued oversupply of teachers, despite the significant constraints imposed following the previous review. The oversupply was attributed to the continuing substantial demand for places: many students move to other jurisdictions to complete their programs, encouraged by arrangements negotiated between Nova Scotia seeking certification. As a consequence, a recent Nova Scotia study of teacher supply and demand predicted that supply will exceed demand by a ratio of close to 3:1 from now until 2014, with approximately 1,000 teachers entering the system every year and only about 360 leaving (Nova Scotia Department of Education, 2007). However, shortages would continue to be felt in specific geographical areas of the province and in some subject areas (but not in mathematics).

Teacher certification in Nova Scotia is administered by the Department of Education. The Office of Teacher Certification is advised by a representative committee of stakeholders appointed by the minister of education.

### NEWFOUNDLAND AND LABRADOR

Newfoundland and Labrador has only one university offering teacher-education programs. A total of approximately 1,000 students register and about 500 graduate annually. The main program divisions are referred to as Primary/Elementary and Intermediate/ Secondary. The Primary/Elementary program is concurrent, requiring a total of five years to complete. Students enter the professional component usually in the third year. The Intermediate/Secondary program is a three-semester consecutive one, completed over 14 months. Specialized programs are available in music, physical education, technology, special education, and other areas. All programs are designed to meet the province's teacher-certification requirements.

Teacher certification in Newfoundland and Labrador is governed by a representative body of stakeholders, and the system is administered by the Department of Education. The most recent comprehensive study of teacher supply and demand in Newfoundland and Labrador was completed in 2001. At that time, Dibbon and Sheppard (2001) found that in spite of a large pool of substitute teachers in the province, the supply of teachers in the specialized areas of mathematics and science was low, and remote communities had difficulty recruiting teachers. Since that time, both the number of new positions and of new graduates per year have increased, suggesting that the supply-demand ratio has not changed very much.

A recent review of school mathematics programs in Newfoundland and Labrador (Atlantic Evaluation and Research Consultants, 2007), which included a comprehensive survey of mathematics teachers, revealed that only a small proportion (about 10 per cent) of elementary school teachers have mathematics as their major area of study. The proportions are much higher for Grade 9 (72 per cent) and for senior-secondary school (83 per cent).

## The administration of the TEDS-M survey in Canada

As stated previously, TEDS-M was administered in four provinces in Canada (Ontario, Quebec, Nova Scotia, and Newfoundland and Labrador). These four provinces mandated the Council of Ministers of Education, Canada (CMEC), to act as the National Research Centre (NRC) responsible for the coordination of the study.

NRC was responsible for representing provinces at international meetings; adapting, translating, and approving all assessment and questionnaire materials; sampling institutions, educators, and future teachers in accordance with international guidelines; contacting participating institutions and sending and collecting materials; arranging for quality-monitoring of the administration; arranging the coding of curriculum documents, questionnaires, and assessment materials; capturing and validating all data; and analyzing the data and publishing the national report.

#### REPRESENTATION AT INTERNATIONAL MEETINGS

The International Study Centre (ISC) for TEDS-M was located at Michigan State University. ISC was responsible for the planning, implementation, and reporting of all aspects of the study at the international level. A conceptual framework (Tatto et al., 2008) guided the design of the study. Throughout the development of the framework and of all instruments, the international group of experts from ISC consulted NRC on a regular basis, including at biannual meetings.

## ADAPTATION, TRANSLATION, AND APPROVAL OF ASSESSMENT AND QUESTIONNAIRE MATERIALS

All materials were developed by ISC in English and approved by all participating countries. Individual countries were responsible for adapting materials for use in their own country and for translating all materials into the target language(s) for the country. In Canada, TEDS-M was administered in English and in French, and all documents (manuals, questionnaires, coding guides) had to be approved by ISC prior to the administration.

#### Sampling institutions

In order to ensure that valid cross-national comparisons could be made, ISC implemented strict sampling guidelines based on national probability samples. In Canada, with only four provinces participating, it was determined that all 31 institutions in these provinces would be invited to participate. However, institutions' participation was voluntary and the institutional response rate was very low (37% or n=10). This could be explained in large part by the fact that the administration of the survey was occurring at the end of April, a very busy period for institutions with end-of-term exams and practica. In several cases, institutions' research-ethics committees/boards did not have sufficient time to review the application to conduct the study.

A total of 94 educators were sampled in the 10 participating institutions. Again, it was determined to invite all educators (of pedagogy, mathematics, and mathematics pedagogy) to participate. Overall, 74 educators responded to the questionnaires, a response rate of 79 per cent.

For the reasons stated above, the response rates for future teachers were also very low. Overall, 183 future teachers at the elementary level and 155 future teachers at the secondary level participated (response rates of 69 per cent and 79 per cent, respectively).

As a result of these low response rates (overall and within institutions), the sampling referee determined that the Canadian sample was not sufficiently representative of the overall population, and ISC elected to report all Canadian results in appendix to the international report and did not include Canada in all international comparisons.

### CONTACTING INSTITUTIONS AND SENDING AND COLLECTING MATERIALS

Administering the TEDS-M survey was a collaborative task shared between the National Research Centre, institutional coordinators, and survey administrators.

Institutional coordinators were responsible for seeking the participation of educators and of future teachers, as well as for most administrative arrangements for the TEDS-M main study at the institution, including providing a list of future teachers and educators, administering educator and institutional questionnaires, and collecting syllabi.

The National Research Centre appointed survey administrators to administer the sessions to future teachers. Survey administrators were selected on the basis of their experience. If possible, individuals who had previous experience in survey or test administration (e.g., the Trends in International Mathematics and Science Study, the Progress in International Reading Literacy Study, the Programme for International Student Assessment) were selected. These survey administrators reported directly to the National Research Centre.

## QUALITY MONITORING

In all participating countries, approximately 10 per cent of participating institutions were visited during the administration of the survey. International quality monitors ensured that the administration procedures were strictly followed and provided detailed reports to ISC.

Generally, the institutional coordinator was responsible for identifying the most suitable individual who could complete the institutional questionnaire. The respondent was asked to complete the questionnaire confidentially and return it to the institutional coordinator. It took approximately two hours to complete this questionnaire.

The institutional coordinator was also responsible for identifying all educators who were teaching to the target population of future teachers at the time of the administration. The questionnaire for educators took approximately 30 minutes to complete.

The questionnaire for future teachers was administered by the survey administrator over a two-hour, uninterrupted period. There were two different versions of the Future Teacher Questionnaire, one for elementary teachers and one for secondary teachers. In each case, several equivalent forms of the questionnaires were used. Three of the four sections were equivalent across all forms of the questionnaires: General Background (5 minutes), Opportunity to Learn (15 minutes), and Beliefs about Mathematics and Teaching (10 minutes). The Mathematics Knowledge for Teaching section (60 minutes) differed across booklets, with some anchor questions to ensure equivalence across forms. Some sample questions can be found in the section below.

## CODING

## Description of teacher-education courses and collection and coding of syllabi

In all participating institutions, the institutional coordinator asked participating educators to provide the course syllabus for each course they were teaching at the time of the data collection. These syllabi were sent to the National Research Centre for coding (see Appendix I for a sample of a coded syllabus).

Syllabus analysis is a systematic method of obtaining comprehensive information from syllabus documentation and materials. It examines content, sequence (when and what topics are intended), and, when appropriate, performance expectations (what students are expected to do with the content) at the institutional and course levels. Qualitative coding protocols were used to capture the courses' academic content.

TEDS-M used a low inference procedure to perform the analysis to reliably investigate the opportunities to learn in different institutions and different countries. In addition to understanding how to code the different syllabi, coders developed a high level of reliability when coding. Inter-coder reliability was closely monitored at the international level. An example of the syllabus-coding activity can be found in Appendix I.

## Coding of mathematics content knowledge and mathematics pedagogical content knowledge

As described in the TEDS-M conceptual framework, teaching mathematics consists of two constructs: mathematics content knowledge and mathematics pedagogical content knowledge (see Appendix II for sample items). Each question in the Mathematics Knowledge for Teaching section measured one of the four content domains (number, geometry, algebra, data) and one of the three cognitive domains (knowing, applying, reasoning). Each question in the Mathematics Pedagogical Content Knowledge section measured one of the three aspects of the framework (mathematical curricular knowledge, knowledge of planning for mathematics teaching and learning, enacting mathematics for teaching and learning).

Coding of all the knowledge questions took place in a central location. All coders were graduating future teachers of mathematics who received extensive training. Coder reliability was closely monitored throughout the coding session. In Canada, all coding took place in both English and French, ensuring a high degree of consistency across languages.

## Results

As mentioned previously, TEDS-M surveyed three populations: institutions, educators, and future teachers. This section presents some selected Canadian results from the three surveys. Due to the very low response rates, it should be noted that these results are descriptive of the participating respondents; generalizations made to the entire population are not advised.

## Institutional Program Questionnaire

Overall, 82 questionnaires were distributed to 10 participating institutions because these offered more than one program. In some cases, institutions offered consecutive and/or concurrent programs; in others, they offered programs at the elementary and/or secondary levels; and in others yet, programs in English and/or French. All these possible variations explain the number of questionnaires analyzed.

Here are some findings from the Institutional Program Questionnaire:

- A majority of institutions (65%) prepare future teachers to teach in both elementary and secondary schools, and there are as many concurrent as consecutive programs represented in the sample.
- Over 60% of institutions require future teachers undertaking the program to have completed at least Grade 12 mathematics (advanced or regular). However, 26% of programs do not require a minimum level of mathematics for future teachers.
- Almost all institutions (87%), within guidelines set by regional or provincial authorities, set policies governing which applicants are admitted to the program.
- When selecting students for their teacher-preparation programs, institutions pay particular attention to candidates' suitability for teaching, which may be assessed by an interview or a written application, or by their demonstrated high level of achievement in mathematics (with 87% and 65% of institutions rating these criteria as somewhat important or very important, respectively). However, candidates' overall achievement level (91%) and performance at the end of the school year (100%) are either not considered or considered not important for selection purposes. This finding would need to be investigated further as it seems to be inconsistent with findings from Crocker and Dibbon (2008) which suggest that institutions do consider minimum averages in a set of prerequisite courses, especially in a competitive environment where the number of applicants exceeds the number of places.
- With respect to provincial norms, respondents view future teachers entering the program as either high- or above-average achievers (44% and 52%, respectively).
- In the programs under study, respondents state that either moderate or major weight is placed on the following program goals:
  - o planning lessons based on recommended pedagogical principles (100%)
  - o learning to reflect on one's own learning and teaching practices (100%)
  - studying the content of the school curriculum and of the mathematics curriculum to be taught (100%)

- No, little, or only some weight is placed on the following goals:
  - o interpreting data from externally conducted tests (69%)
  - studying mathematics or other disciplines at the tertiary level (65% and 69%, respectively)
  - o learning specific strategies and curricula for teaching gifted students (61%)
  - o developing the knowledge and skills to do teacher action research (70%)
- In terms of institutional requirements for successfully completing the program, all respondents (100%) state that future teachers must pass all courses required for the program, successfully demonstrate a required level of teaching competence in the classroom, and receive a passing grade on their field experience. None of the programs require future teachers to write a thesis, and only a few (less than 10%) require future teachers to pass a written or an oral examination.
- In about three-quarters of the programs, teachers who mentor future teachers during their extended field practice receive some form of compensation. As part of their responsibilities, these mentors are expected, in all programs, to observe future teachers in classrooms. They are also expected to provide a formal summative assessment and written narrative reports of the field practice.
- Generally, institutions are responsible for finding placements for the extended teaching practice (in 82% of programs), and although about half of the institutions have no difficulty finding suitable placements for future teachers, another 30% state that appropriate placements are not readily available in sufficient numbers.
- Of the faculty employed to teach mathematics or mathematics-related content to future teachers, all institutions indicate that a master's degree would normally be required and 70% of institutions would normally require a doctorate. Few institutions require that these educators be qualified to teach or have experience teaching at the elementary or secondary level. However, educators teaching at the elementary or secondary level to have some experience teaching at the elementary level in about half of the programs. It should be noted that this relative lack of a requirement for teaching experience in some programs does not mean that many educators do not have some teaching experience. This is illustrated below by the fact that more than half of the surveyed educators held a teaching certificate. All institutions require that mentors supervising the extended teaching practice hold at least a bachelor's degree.

## Survey for Mathematics, Mathematics Pedagogy, and General Pedagogy Educators

Overall, 194 questionnaires were distributed and 94 questionnaires were returned.

• The academic rank of the respondents was well distributed across all possible ranks, with 19% being professors, 16% being associate professors, 23% assistant professors, 17% lecturers, 14% instructors, and 11% in other roles. There were slightly more females (56%) than males.

- Of these educators, 27% had a degree in education, 14% in mathematics education, and 10% in mathematics at the doctoral level or higher. These figures should be interpreted with caution. As stated above, a large number of respondents to the survey were likely to be sessional appointees.
- Fifty-two per cent of respondents held a teaching certificate at the time of the study; one-third has never held a teaching certificate.
- On average, these educators have spent approximately 10 years at their current institutions and about seven years in their teacher-preparation role. They teach mostly at the undergraduate level (93%), but also at the master's level (48%), and, in some cases, at the doctoral level (18%). In terms of experience, 30% have been involved in instructing or supervising future teachers for over 10 years, but 37% have done this for three years or less, suggesting the existence of a cadre of educators with relatively little experience, at least when considering the general profile of the respondents.
- Over the past 12 months, more than half of these educators have not participated in any professional-development activities related to mathematics or mathematics pedagogy. However, over 60% had participated in at least six hours of professional development in general pedagogy.
- In the past year, respondents have spent about 30% of their working time on research and research-related activities.
- When asked to consider all courses in the teacher-education programs offered at their institutions, about 80% of these educators felt that each stage seemed to be planned to meet the main needs of future teachers at each stage of their preparation, and that the programs were organized in a way that covered what future teachers needed to become effective teachers. However, 30% disagreed that there were clear links between most of the courses in the program of study.
- When asked to describe future teachers' level of preparedness for teaching mathematics as they complete the teacher-education program and start their teaching careers, 80% or more of these educators felt that future teachers were prepared to a moderate or major extent to clearly communicate ideas and information about mathematics to students; that they could establish appropriate learning goals in mathematics for students; and that they could set up mathematics learning activities to help students achieve learning goals. Conversely, over a third of these educators felt that future teachers were not at all or just to a minor extent prepared to use computers and information and communication technology (ICT) to aid in teaching mathematics and to provide parents with useful information about students' progress in mathematics. Further research would be useful to compare the views of teachers, principals, and faculties regarding the level of preparedness in using ICT to teach mathematics.
- Finally, almost 80% of educators felt that their pre-service teacher-education programs were effective or very effective in preparing future teachers of mathematics.

## Survey for Future Teachers of Lower-Secondary Mathematics

Overall, 282 questionnaires were distributed to future teachers at the secondary level and 155 were completed.

- The median age of respondents was 24 years, and 61% were females. About 86% of respondents always or almost always spoke the language of the questionnaire at home. Eighty-three per cent had taken a calculus or pre-calculus course in high school. When comparing themselves to other students at the same grade level in their schools, 82% saw themselves as being at the top or near the top of their class in terms of marks.
- When asked to identify the main reasons why they decided to become teachers, these future teachers at the secondary level mentioned their perception of teaching as a challenging job, their desire to have an influence on the next generation, their desire to work with young people, their belief that they have a talent for teaching, and their love of mathematics as the most significant reasons. For the most part, they were not influenced by teacher salaries, the availability of teaching positions, or the fact that they had always been good students in school.
- Almost half (47%) of these future teachers said that needing to borrow money hindered their studies during their teacher-preparation programs.
- Almost all future teachers at the secondary level (95%) thought that they would or could possibly spend their entire career as a teacher.
- These respondents were asked to consider a number of mathematics topics and to indicate which ones they had or had never studied. Over 80% stated that they had studied calculus, linear algebra, probability, and theoretical or applied statistics. Fifty per cent or more stated that they had never studied topology, differential geometry, theory of real or complex functions or functional analysis, or abstract algebra. In the context of their teacher-education programs, more than half also said that they had not studied validation, structuring and abstracting, or calculus. In terms of emphasis, three-quarters said that in their programs, they studied mathematics at the level of the school curriculum.
- In terms of mathematics pedagogy, over 80% said that their programs covered mathematics instruction, mathematics standards and curriculum, and the development of teaching plans. Their programs allowed them to engage in the following activities most often: accommodating a wide range of abilities in each lesson; exploring how to use manipulatives; and creating learning experiences that make central concepts under study meaningful to students. The activities that were mentioned the least often were: using standardized tests to guide decisions about what and how to teach; using assessment to give feedback to parents or guardians; and helping students learn how to assess their own learning.
- When looking at education and pedagogy topics, over a third of the secondary-level future teachers said that they had not studied methods of educational research or the history of education and educational systems.

- With regard to their practica, over 60% agreed that the feedback they received from their mentors helped them to improve their teaching methods; that this feedback helped them to improve their understanding of their students; and that they had a clear understanding of what their mentors expected of them as teachers in order to pass the practica.
- Regarding the perceived overall effectiveness of their pre-service teacher-education programs, almost a quarter of these future teachers at the secondary level felt that the programs were not effective in preparing them to teach mathematics.

## Survey for Future Teachers of Elementary Mathematics

Overall, 580 questionnaires were distributed to future teachers at the elementary level and 183 were completed. Although the number of respondents is quite substantial, the response rate is below the international standard; therefore, any generalization to the population of future elementary teachers should be made with extreme caution.

- Eighty-seven per cent of respondents of this questionnaire were female. This may be slightly higher than the overall population of elementary teachers. Based on the Progress in International Reading Literacy Study (2006) data, in which five provinces participated, the figures would vary between 72% and 86% for female teachers at the elementary level. Unfortunately, no official data are available at the pan-Canadian level for this aspect.
- About 95% always or almost always spoke the language of the questionnaire at home. Only 40% had taken a calculus or pre-calculus course in high school, which is much lower than for future teachers at the lower-secondary level. When comparing themselves to other students at the same grade level in their schools, about half of the respondents saw themselves as being at the top or near the top of their class in terms of marks, again a much lower percentage than for the lower-secondary level.
- When asked to identify the main reasons why they decided to become teachers, these future teachers at the elementary level had very similar response patterns to those at the lower-secondary level. They mentioned their talent for teaching, their perception of teaching as a challenging job, and their desire to work with young people and to have an influence on the next generation as the most significant reasons. For the most part, they were not influenced by teacher salaries, the availability of teaching positions, or their love of mathematics (which is logical since elementary-level teachers are normally generalists).
- These respondents were asked to consider a number of mathematics topics and to indicate which ones they had or had never studied. As can be expected, the vast majority had not studied many advanced topics in mathematics, with over 90% not having studied advanced calculus, abstract or linear algebra, or theory of real or complex functions or functional analysis. In fact, the only advanced topics

studied by more than half of these respondents were probability and number theory. In the context of their teacher-education programs, over 85% said that they had not studied calculus, validation, or structuring and abstracting. In terms of emphasis, almost 90% said that in their programs, they studied mathematics at the level of the school curriculum, and 63% said that their learning was at a deeper or more conceptual level than that of the school curriculum.

- In terms of mathematics pedagogy, over 80% said that their programs covered mathematics instruction, mathematics standards and curriculum, and mathematics teaching per se. Their programs allowed them to engage in the following activities most often: exploring how to use manipulatives; creating learning experiences that make central concepts under study meaningful to students; locating suitable curriculum materials and teaching resources and developing games or puzzles; accommodating a wide range of abilities in each lesson; and building on students' existing mathematics knowledge and thinking skills. The activities that were mentioned the least often were: using standardized tests to guide decisions about what and how to teach; using assessment to give feedback to parents or guardians; and helping students learn how to assess their own learning (similar to lower-secondary respondents).
- When looking at education and pedagogy topics, over a third of the elementarylevel future teachers said that they had not studied methods of educational research or the history of education and educational systems.
- With regard to their practica, as was the case for the lower-secondary level, over 60% agreed that they had a clear understanding of what their mentors expected of them as teachers in order to pass the practica; that the feedback they received from their mentors helped them to improve their teaching methods; and that this feedback helped them to improve their understanding of their students. A large proportion of these respondents also felt that their mentors valued the ideas and approaches they brought from their teacher-education programs.
- Regarding the perceived overall effectiveness of their pre-service teacher-education programs, elementary-level future teachers were generally more pleased than secondary-level future teachers with their programs in preparing them to teach mathematics.

An important contribution of TEDS-M is the assessment of participating future teachers with regard to their knowledge of mathematics, of mathematics pedagogy, and of general pedagogy. The design of TEDS-M lends itself to the ranking of countries according to their mean scores on these three areas. However, international comparisons based on these rankings are not desirable because these mean scores are relatively unstable, due to the small number of respondents in several participating countries. Moreover, since Canada did not meet the sampling requirements, the Canadian data must be treated with extreme caution. For the same reason, no results are provided by province, although ministries and departments of education have access to their own data in order to analyze their results according to their own contexts. What may be most useful in the Canadian context is to look at countries that tend to perform better on this knowledge component and to analyze their contextual factors, which may help to explain their results. Furthermore, it should be kept in mind that teachereducation programs in Canada are far from homogeneous — even within a province — and that it may be more useful to analyze the contextual factors in each province, taking into account the limitations of the study.

Results are presented separately for teachers at the elementary level and teachers at the lower-secondary level, and are broken down for mathematics content knowledge and mathematics pedagogical content knowledge. Results are scaled, with an international mean of 500 and a standard deviation of 100.

### Figure 1 Mean scores and confidence intervals for participating countries— Elementary-level future teachers' mathematics content knowledge



Note: The Canadian average score is not part of the international average score due to Canada's low response rates.



#### Figure 2 Mean scores and confidence intervals for participating countries— Elementary-level future teachers' pedagogy content knowledge

Note: The Canadian average score is not part of the international average score due to Canada's low response rates.

Among the other 15 countries participating in the study, elementary-level future teachers from Chinese Taipei and from Singapore outperformed those from other countries in both mathematics content knowledge and mathematics pedagogical content knowledge. Another group of countries, comprising Switzerland, Norway, and the United States, had results above the international average in both mathematics and pedagogical content knowledge, as did Canada. A group of six countries (Georgia, Chile, the Philippines, Botswana, Spain, and Poland) performed below the international average in both components.

#### Figure 3 Mean scores and confidence intervals for participating countries— Lower-secondary-level future teachers' mathematics content knowledge



Note: The Canadian average score is not part of the international average score due to Canada's low response rates.



### Figure 4 Mean scores and confidence intervals for participating countries— Lower-secondary-level future teachers' pedagogy content knowledge

Note: The Canadian average score is not part of the international average score due to Canada's low response rates.

The results are quite similar at the lower-secondary level; however, future teachers from Chinese Taipei significantly outperformed all other countries in both mathematics and pedagogical content knowledge. This is not surprising considering that on the PISA 2006 mathematics assessment, 15-year-old students from Chinese Taipei outperformed students from all other countries. Interestingly, future teachers from the Russian Federation had a much stronger performance in both aspects at the lower-secondary level compared to the elementary level. This also holds true for German future teachers, although to a lesser extent. Conversely, future teachers from Norway performed below the international average at the lower-secondary level, even though they performed above the international average at the elementary level.

Keeping in mind the serious data limitations for Canada, it is worth noting that our future teachers performed above the international average in both mathematics and pedagogical content knowledge at both the elementary and lower-secondary levels.

Figure 5 Mean scores and confidence intervals by gender: primary mathematics knowledge



Figure 6 Mean scores and confidence intervals by gender: primary pedagogical knowledge



Among Canadian future teachers at the elementary level, there was no statistical difference between their mathematics and pedagogical content knowledge. Although the mean score for female future teachers was higher than that for males in pedagogical content knowledge, the difference was not statistically significant. Of note is the large imbalance in the proportion of male teachers in the sample (1 in 7). However, this is a fairly accurate reflection of the situation in the population, which sees about 80% of teachers in elementary schools in Canada being female (Mullis et al. 2007).


# Figure 7 Mean scores and confidence intervals by gender: lower-secondary mathematics knowledge





At the lower-secondary level, male future teachers had higher mean scores in both mathematics and pedagogical content knowledge than did females, but the differences were not statistically significant. The gender imbalance in the sample was not as pronounced at this level as at the elementary level (three female future teachers for every two males at the lower-secondary level). Overall, in Canada, this is a fair reflection of the situation at the lower-secondary level, which sees approximately 70% of teachers being female.<sup>2</sup>

<sup>&</sup>lt;sup>2</sup> According to the Pan-Canadian Assessment Program 2007, across all provinces and territories, between 50% and 87% of teachers of 13-year-olds are females.

# Conclusion

TEDS-M is a comparative study of teacher education, with a focus on the preparation of mathematics teachers at the elementary and lower-secondary levels. As the first attempt to systematically investigate both the knowledge and skills of future teachers in an international setting, the study faced significant challenges in several countries. In Canada, given the structure of our education systems, provinces and territories individually determined whether they wished to be involved in such an endeavour. Four provinces (Ontario, Quebec, Nova Scotia, and Newfoundland and Labrador) elected to participate, and in each province, the participation of individual institutions, individual educators, and individual future teachers was completely voluntary. The very low response rates obtained in Canada and the ensuing non-inclusion in the international data set are indeed disappointing — but also very informative — should such an initiative be repeated in the future. Conducting such an ambitious assessment project is possible, but would require engaging all parties early on and throughout the implementation of the project.

We are grateful to the ministries/departments of education, institutions, faculties of education, educators, and future teachers who did participate in this TEDS-M 2008 assessment project. It is hoped that the project will yield useful information to guide teacher education in the future.

Keeping in mind the serious limitations of the data gathered from TEDS-M, there are a few conclusions from the study that may help elicit further discussion among policymakers and educators:

- In general, Canadian institutions do not seem to have very strong requirements in mathematics for individuals wishing to enter into teacher-education programs. Other than assessments to meet the requirements for a course in mathematics or in mathematics education, there is no formal test of the mathematical knowledge or skills of future teachers at the completion of their program of study.
- Most educators of mathematics or of mathematics pedagogy for future teachers in Canadian universities are specialized in areas other than mathematics, and few hold a doctorate in the discipline. At the international level, about one-quarter of educators who participated in the TEDS-M study held a doctorate in mathematics, with a higher proportion in higher-performing countries. In Canada, 10 per cent of participating educators held a doctorate in mathematics.
- With regard to knowledge of mathematics and of mathematics pedagogy, Canadian future teachers at the elementary and lower-secondary levels performed above the international average. Although Canadian 15-year-olds have performed among the top-ranking countries in mathematics since the inception of PISA in 2000, it appears that there is potential for further improvement by possibly providing future teachers at the elementary level with more advanced training in mathematics that covers elements beyond the school curriculum, and by providing future teachers at the lower-secondary level with more advanced training in testing- and assessment-related topics.

# References

Atlantic Evaluation and Research Consultants. (2007). K-12 mathematics curriculum review: Final report. Retrieved from http://www.ed.gov.nl.ca/edu/publications/k12/ NLMathReport.pdf

Blömeke, S., Kaiser, G., & Lehmann, R. (Eds.). (2010). *TEDS-M 2008. Professional Competences and Opportunities to Learn of Future Mathematics Teachers at Lower-Secondary Schools in the International Context.* Münster: Waxmann.

Blömeke, S., Kaiser, G., & Lehmann, R. (Eds.). (2010). *TEDS-M 2008. Professional Competences and Opportunities to Learn of Future Primary Teachers in the International Context.* Münster: Waxmann.

Carnoy, M., Beteille, T., Brodziak, I., Loyalka, P., & Luschei, T. (2009). *Do countries paying teachers higher relative salaries have higher student mathematics achievement?* Palo Alto, CA: Stanford University. Retrieved from https://teds.educ.msu.edu/TEDS\_CostsStudy.zip

Council of Ministers of Education, Canada. (2009). *PCAP-13 2007 Contextual Report on Student Achievement in Reading*. Toronto: Author.

Crocker, R. & Dibbon, D. (2008). *Teacher education in Canada: a baseline study.* Kelowna, BC: Society for the Advancement of Excellence in Education.

Dibbon, D. & Sheppard, B. (2001). *Teacher demand, supply and retention in Newfoundland and Labrador*. Memorial University of Newfoundland. Retrieved from http://www.mun. ca/educ/people/faculty/ddibbon/pdf/Final%20report/chapter1-Introduction.pdf

Feng-Jui, H., Ting-Ying, W., Chia-Jui, H., Shu-Jyh, T., Guoheng, C., Chiu-Keung, L., et al. (2010). *A milestone of an international study in Taiwan teacher education*. National Research Center of Taiwan Teacher Education and Development Study in Mathematics in Taiwan.

Ministère de l'Éducation du Québec. (2004). Surplus ou pénurie de personnel enseignant qualifié au Québec : situation actuelle et perspectives à court et moyen terme pour le secteur francophone. Retrieved from http://www.mels.gouv.qc.ca/dftps/interieur/pdf/pers\_emploi\_ prevision\_nov04.pdf

Mullis, I.V.S., Martin, M.O., Kennedy, A.M., & Foy, P. (2007). *PIRLS 2006 International Report.* Chestnut Hill, MA: Boston College.

Mullis, I., Martin, M., Ruddock, G., O'Sullivan, C., Arora, A., & Erberber, E. (2005). *TIMSS 2007 assessment frameworks. International Association for the Evaluation of Educational Achievement.* Retrieved from http://timss.bc.edu/TIMSS2007/PDF/T07\_AF.pdf

Nova Scotia Department of Education. (2007). *Nova Scotia public education teacher supply and demand*. Retrieved from http://www.ednet.ns.ca/pdfdocs/reports/teacher\_supply\_and\_demand\_report.pdf

Nova Scotia Department of Education. (2008). *Report and recommendations of the review panel on teacher education in Nova Scotia*. Retrieved from http://www.ednet.ns.ca/events/teacher\_education\_review/documents/review\_panel\_report\_web\_e.pdf

Ontario College of Teachers. (2001). *Teacher supply and demand: Report on the consultation*. Retrieved from http://www.oct.ca/publications/pdf/tsd\_e.pdf

Ontario College of Teachers. (2007). *State of the teaching profession 2007: Annual Ontario College of Teachers survey of teachers*. Retrieved from http://www.oct.ca/publications/PDF/survey07\_e.pdf

Ontario College of Teachers. (2009). *Transition to teaching 2009*. Retrieved from http://www.oct.ca/publications/PDF/transitions09\_e.pdf

Statistics Canada. (2008). CANSIM. Table 477-0014 — University degrees, diplomas and certificates granted, by program level, Classification of Instructional Programs, Primary Grouping (CIP\_PG) and sex, annual (number). Retrieved from http://dc2.chass.utoronto.ca

Tatto, M.T., Ingvarson, L., Schwille, J., Peck, R., Senk, S.L., & Rowley, G. (2008). *Teacher Education and Development Study in Mathematics (TEDS-M): Policy, practice, and readiness to teach primary and secondary mathematics. Conceptual framework.* East Lansing, MI: TEDS-M International Study Center at Michigan State University. Retrieved from https://teds.educ.msu.edu/TEDS\_FrameworkFinal.zip

# Sample Syllabus Coding



# Math 250: Probability and Statistics for Elementary/ Middle-School Teachers

| Instructor:   | Dr. K.P. Lee                                 | Office: | 456 Honors Tower    |
|---------------|--|---------|---------------------|
| Phone:        | 789-0123                                     | E-mail: | kplee@standardu.edu |
| Office hours: | 3:50–4:50 p.m. on Mondays and by appointment |         |                     |

### COURSE PREREQUISITES

Completion of Math 125 or equivalent, with a C grade or higher. Those not meeting the course prerequisites will be automatically dropped from the course by the Department of Mathematics.

### COURSE MATERIALS

Text: Probability and Statistics for Elementary/Middle-School Teachers

A **TI-73** graphing calculator (other models such as the TI-83 and TI-86 will **not** have the features needed for this course and will not be sufficient)

### COURSE OVERVIEW AND OBJECTIVES

This course is designed to provide pre-service elementary and middle-school teachers with:

| B1 / T2 |
|---------|
| 1.1.7   |
| 3.3.1   |
| 3.3.2   |

[laboratory-based experiences designed to teach and reinforce basic statistics and probability concepts, often situated in the core tasks of teaching, including the examination and critiquing of the statistical thinking and arguments of others;]

| B2 / T2 |
|---------|
| 1.1.7   |
|         |

[opportunities to demonstrate their understanding of statistics and probability concepts in meaningful, constructive, and alternative ways;]

|     | B3 / T2 |  |
|-----|---------|--|
|     | 1.1.7   |  |
|     | 3.5.5   |  |
|     | 3.5.6   |  |
| 1.0 |         |  |

[activities involving the use of calculator technology (this course is used to satisfy part of the University Computer Usage requirement for students in elementary teacher education);]

[experiences that model techniques for use in their future elementary/ middle-school classrooms.]



[The majority of class time will be devoted to small-group exploration and class discussion, with an emphasis on reasoning and communication of statistics and probability ideas rather than on rote memorization. Students will be expected to demonstrate that they are prepared for class each day by bringing course materials and *completed homework assignments*. Students will be asked to work cooperatively in groups and participate in whole-class discussions. Students' consistent engagement in these activities will be reflected in their participation grade.]

## Codes Used in this Excerpt

| U1:        | Sequential unit number of the coded document   |
|------------|--|
| T2:        | An objective of the course   |
| T2.4:      | A course that covers specialized mathematics knowledge needed for teaching mathematics |
| B1, B2, B3 |  |
| and B4:    | Each block of information being coded  |
| 1.1.7:     | Mathematics block on data representation, probability, and statistics                  |
| 3.3.1:     | Mathematics education block on developing mathematical concepts                        |
| 3.3.2:     | Mathematics education block on reasoning, argumentation, and proving                   |
| 3.5.5:     | Mathematics instruction block on using calculators                                     |
| 3.5.6:     | Mathematics instruction block on using computers                                       |

Examples of assessment items in the Primary General Pedagogy Knowledge for Teaching section of the Future Teacher Questionnaire

#### Example 1:

Label: MFC106 Item type: Multiple choice Dimension: Mathematics Content Knowledge Domain: Data Sub-domain: Applying

Two fair six-sided number cubes are thrown in a probability game and the two numbers at the top are recorded.



Josie wins if the difference between the two numbers is 0, 1 or 2. Farid wins if the difference between the two numbers is 3, 4 or 5.

The students discuss whether the game is fair.

Which of the following statements is correct?



For this probability question, only half of the future teachers responded correctly that Josie had a greater chance of winning. One-third of the respondents were incorrectly drawn to the first option, thinking that both Josie and Farid had an equal chance of winning.

#### Example 2:

| Label: MFC108  |
|--|
| Item type: Multiple choice                           |
| Dimension: Mathematics Pedagogical Content Knowledge |
| Domain: Algebra                                      |
| Sub-domain: Enacting                                 |

Amy is building a sequence of geometric figures with toothpicks by following the pattern shown below. Each new figure has one extra triangle.

Variable *t* denotes the position of a figure in the sequence.



In finding a mathematical description of the pattern, Amy explains her thinking by saying: "I use three sticks for each triangle."



"Then I see that I am counting one stick twice for each triangle, except the last one, so I have to remove those."

Variable *n* represents the total number of toothpicks used in a figure.

Which of the equations below best represent Amy's statement in algebraic notation?

|    |                  | Check <u>one</u> box. |
|----|------------------|-----------------------|
| А. | n = 2t + 1       | .29                   |
| В. | n=2(t+1)-1       | .26                   |
| C. | n = 3t - (t - 1) | .36                   |
| D. | n = 3t + 1 - t   | 08.                   |

This question was difficult for Canadian future teachers, with almost two-thirds of the respondents unable to identify the correct algebraic notation for this pattern.

#### Example 3:

| Label: MFC202                            |
|--|
| Item type: Multiple choice               |
| Dimension: Mathematics Content Knowledge |
| Domain: Algebra                          |
| Sub-domain: Knowing                      |

Indicate whether each of the following statements is true for the set of all whole numbers a, b and c greater than zero.

|                                | Check <u>one</u> box in <u>each</u> row. |          |
|--------------------------------|--|----------|
|                                | True                                     | Not True |
| A. $a-b=b-a$                   | .16                                      | .84      |
| B. $a \div b = b \div a$       | .06                                      | .95      |
| C. $(a + b) + c = a + (b + c)$ | .93                                      | .07      |
| D. $(a-b) - c = a - (b-c)$     | .23                                      | .77      |

This algebra question was relatively easy for Canadian future teachers. However, almost one-quarter of the respondents did not know that the fourth statement was not true, even though the question appeared to be relatively easy to solve with whole numbers.

#### Example 4:

Label: MFC203 Item type: Multiple choice Dimension: Mathematics Content Knowledge Domain: Geometry Sub-domain: Applying

A rectangular-shaped swimming pool has a paved walkway (shaded) around it as shown.



not to scale

What is the area of the walkway?

|    |                     | Check <u>one</u> box. |
|----|---------------------|-----------------------|
| А. | 100 m <sup>2</sup>  | .04                   |
| В. | 161 m <sup>2</sup>  | .06                   |
| C. | $710 \text{ m}^2$   | .78                   |
| D. | 1610 m <sup>2</sup> | .12                   |

This question was relatively easy for Canadian respondents, with 78% getting the correct answer. However, 12% calculated the entire area but failed to subtract the area of the swimming pool.

#### Example 5:

| Label: MFC206                            |
|--|
| Item type: Constructed response          |
| Dimension: Mathematics Content Knowledge |
| Domain: Number                           |
| Sub-domains: Applying, Planning          |

a) A machine uses 2.4 litres of fuel for every 30 hours of operation. How many litres of fuel will the machine use in 100 hours if it continues to use fuel at the same rate?

|    |     | Check <u>one</u> box. |
|----|-----|-----------------------|
| А. | 7.2 | .11                   |
| В. | 8.0 | <b>I</b> .79          |
| C. | 8.4 | .08                   |
| D. | 9.6 | .02                   |

This question was relatively easy for Canadian respondents, with almost 80% getting the correct response. However, one in five respondents did make a calculation error when solving this simple problem.

b) Create a different problem of the same type as the problem in (a) (same processes/ operations) that is **EASIER** for primary children to solve.

# Scoring Guide

| Code | Response   | Item: MFC206B              |  |  |
|------|--|----------------------------|--|--|
|      | Correct Response   |                            |  |  |
| 10   | A different problem, of the <b>same type</b> (same pro <b>easier</b> to solve.   | cesses/operations), but    |  |  |
|      | Examples:  |                            |  |  |
|      | • A machine uses 3 litres of fuel for every 30 hours of operation. How many litres of fuel will the machine use in 100 hours?                                  |                            |  |  |
|      | • A car uses 2.4 litres of fuel for every 50 km. How many litres of fuel will the car use in 100 km?   |                            |  |  |
|      | Incorrect Response   |                            |  |  |
|      | A different problem, of the <b>same type</b> (same pro<br><b>NOT easier</b> to solve (Note: Items judged to be o<br><b>difficulty</b> are <b>NOT easier</b> .) |                            |  |  |
|      | Examples:  |                            |  |  |
| 70   | • A machine uses 2 litres of fuel for every 30 hours of operation. How many litres of fuel will the machine use in 100 hours? (2 is not divisible by 3)        |                            |  |  |
|      | • A tap drips 2 litres of water every day. How m<br>(The metric knowledge required and the<br>significantly higher.)   | · ·                        |  |  |
| 79   | Other incorrect response (including crossed out, task responses, and stray marks)  | erased, illegible, or off- |  |  |
|      | Examples:  |                            |  |  |
|      | • questions that are not meaningful or have no   | answer                     |  |  |
|      | No Response  |                            |  |  |
| 99   | Blank  |                            |  |  |

About 84% of Canadian future teachers were able to create a problem of the same type but that was easier to solve. Nine per cent of respondents created a similar problem but that was not easier to solve.

Examples of assessment items in the Secondary General Pedagogy Knowledge for Teaching section of the Future Teacher Questionnaire

Example 1:

| Label: MFC604                            |
|--|
| Item type: Constructed response          |
| Dimension: Mathematics Content Knowledge |
| Domain: Algebra                          |
| Sub-doman: Applying                      |

The following problems appear in a mathematics textbook for lower secondary schools.

- 1. Peter, David, and James play a game with marbles. They have 198 marbles altogether. Peter has 6 times as many marbles as David, and James has 2 times as many marbles as David. How many marbles does each boy have?
- 2. Three children Wendy, Joyce, and Gabriela have 198 zeds altogether. Wendy has 6 times as much money as Joyce, and 3 times as much as Gabriela. How many zeds does each child have?

(a) Solve each problem.

Solution to Problem 1:

Solution to Problem 2:

#### **Scoring Guide**

**Note:** The correct answers to MFC604A1 and MFC604A2 are as follows:

Problem 1: David has 22 marbles, Peter has 132 marbles, and James has 44 marbles.

Problem 2: Wendy has 132 zeds, Joyce has 22 zeds, and Gabriela has 44 zeds.

The following methods are considered in the scoring guide:

1) Using one variable, setting up one equation, and solving.

*Example* (Problem 1): Let m = the number of marbles that David has. Then Peter has 6m and James has 2m. Therefore, 6m + 2m + m = 198, and m = 22.

2) Using *more than one variable*, establishing a *system of equations*, performing substitutions, and solving.

*Example* (Problem 1): Let p = the number of marbles that Peter has, d = the number of marbles that David has, and j = the number of marbles that James has. Therefore, p = 6d and j = 2d, p + d + j = 198.

- 3) Trial and error or guess and check
- 4) Ratio or other arithmetic methods
- 5) Representation/diagram

| Code | Response   | Item ID: MFC604A1              |  |
|------|--|--------------------------------|--|
|      | Correct Response   |                                |  |
| 11   | Response uses Method 1 correctly to solve Problem 1 and get the correct answers.   |                                |  |
| 12   | Response uses Method 2 correctly to solve Problem  | 1 and get the correct answers. |  |
| 13   | Response uses Method 3 correctly to solve Problem  | 1 and get the correct answers. |  |
| 14   | Response uses Method 4 correctly to solve Problem  | 1 and get the correct answers. |  |
| 15   | Response uses Method 5 correctly to solve Problem  | 1 and get the correct answers. |  |
| 19   | Response uses a correct but different method from those listed above to solve<br>Problem 1 and get the correct answers.  |                                |  |
|      | Incorrect Response   |                                |  |
| 70   | Response uses one of Methods 1 to 5 to solve Problem 1, but arrives at an incorrect answer or cannot complete the solution because of a computation or algebra error.  |                                |  |
| 71   | Response uses a correct but different method from those listed above to solve<br>Problem 1, but arrives at an incorrect answer or cannot complete the solution<br>because of a computation or algebra error. |                                |  |
| 79   | Other incorrect response (including crossed out, erased, illegible, or off-task responses, and stray marks)  |                                |  |
|      | No Response  |                                |  |
| 99   | Blank  |                                |  |

| Code | Response Item I   | D: MFC604A2                                |  |  |
|------|---|--|--|--|
|      | Correct Response  |  |  |  |
| 11   | Response uses Method 1 to solve Problem 2.  |  |  |  |
| 12   | Response uses Method 2 to solve Problem 2.  |  |  |  |
| 13   | Response uses Method 3 to solve Problem 2.  |  |  |  |
| 14   | Response uses Method 4 to solve Problem 2.  | Response uses Method 4 to solve Problem 2. |  |  |
| 15   | Response uses Method 5 to solve Problem 2.  | Response uses Method 5 to solve Problem 2. |  |  |
| 19   | Response uses a correct but different method from those listed above to solve Problem 2 and get the correct answers.  |  |  |  |
|      | Incorrect Response  |  |  |  |
| 70   | Response uses one of Methods 1 to 5 to solve Problem 2, but arrives<br>at an incorrect answer or cannot complete the solution because of a<br>computation or algebra error.     |  |  |  |
| 71   | Response uses a correct but different method from those<br>to solve Problem 2, but arrives at an incorrect answer or<br>complete the solution because of a computation or algel | cannot                                     |  |  |
| 79   | Other incorrect response (including crossed out, erased, off-task responses, and stray marks)   | illegible, or                              |  |  |
|      | No Response   |  |  |  |
| 99   | Blank   |  |  |  |

Overall, Problem 1 was easy to solve for Canadian future teachers, with 85% getting the correct response. The majority of respondents (53%) used Method 1 to solve Problem 1, and 29% used Method 2. Problem 2 was of medium difficulty for Canadian future teachers, with 65% getting a right answer. Sixteen per cent of future teachers used one of the five methods listed to solve Problem 2 but made a computation or algebra error (code 70).

#### Example 2:

| Label: MFC610                            |
|--|
| Item type: Multiple choice               |
| Dimension: Mathematics Content Knowledge |
| Domain: Number                           |
| Sub-domain: Knowing                      |

Determine whether each of the following is an irrational number always, sometimes or never.

|  | Check <u>one</u> box in <u>each</u> row. |           |              |  |
|--|--|-----------|--------------|--|
|  | Always                                   | Sometimes | Never        |  |
| A. The result of dividing the circumference of a circle by its diameter. | <b>⊠</b> .65                             | .26       | 09. 🗖        |  |
| B. The diagonal of a square with side of length 1.                       | <b>☑</b> .75                             | .11       | <b>.</b> 14  |  |
| C. Result of dividing 22 by 7.   | .66                                      | .06       | <b>I</b> .28 |  |

Canadian future teachers had some difficulty identifying the characteristics of irrational numbers. Only 28% of respondents knew that dividing 22 by 7 never results in an irrational number, possibly confusing this fraction with  $\pi$ , which is irrational.

#### Example 3:

| Label: MFC704                            |
|--|
| Item type: Constructed response          |
| Dimension: Mathematics Content Knowledge |
| Domain: Geometry                         |
| Sub-domain: Applying                     |

On the figure, *ABCD* is a parallelogram,  $\angle BAD = 60^\circ$ , *AM* and *BM* are angle bisectors of angles *BAD* and *ABC* respectively. If the perimeter of *ABCD* is 6 cm, find the sides of triangle *ABM*.

Write your answers on the lines below.



#### **Scoring Guide**

| Code | Response   | Item ID: MFC704 |  |  |
|------|--|-----------------|--|--|
|      | Correct Response   |                 |  |  |
|      | Response indicates all three correct entries below:  |                 |  |  |
|      | AB = 2  cm   |                 |  |  |
| 20   | $AM = \sqrt{3}$ cm or equivalent   |                 |  |  |
|      | BM = 1  cm   |                 |  |  |
|      | Partially Correct Response   |                 |  |  |
| 10   | Response indicates any two correct entries and one incorrect (or blank) entry  |                 |  |  |
| 11   | Response indicates any one correct entry and two incorrect (or blank) entries  |                 |  |  |
|      | Incorrect Response   |                 |  |  |
| 79   | Incorrect mathematical statements or statements of no value (including crossed out, erased, illegible, or off-task responses, and stray marks) |                 |  |  |
|      | No Response  |                 |  |  |
| 99   | Blank  |                 |  |  |

This was a difficult question for Canadian future teachers, with only 41% giving all three correct responses. Another 14% had two correct responses, and 8% had one correct response. This means that 37% of respondents could not find any of the three measures of the triangle.

#### Example 4:

| Label: MFC705                            |
|--|
| Item type: Multiple choice               |
| Dimension: Mathematics Content Knowledge |
| Domain: Geometry                         |
| Sub-domain: Knowing                      |

We know that there is only one point on the real line that satisfies the equation 3x = 6, namely x = 2.

Suppose now that we consider this same equation in the plane, with coordinates x and y, and then in space with coordinates x, y, and z. What does the set of points that satisfy the equation 3x = 6 look like in these settings?

|    |                                       | Check <u>one</u> box in <u>each</u> row. |          |              |       |
|----|---------------------------------------|--|----------|--------------|-------|
|    |                                       | One point                                | One line | One plane    | Other |
| А. | The solution to $3x = 6$ in the plane | .22                                      | .69      | 08.          | .01   |
| В. | The solution to $3x = 6$ in space     | .14                                      | 09. 🗖    | <b>I</b> .71 | .07   |

This geometry question was moderately difficult for Canadian future teachers, with about 70% of respondents knowing that the proposed solution would look like one line in the plane and one plane in space.